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SOME NEW CLASSES OF SETS AND DECOMPOSITION OF MICRO CONTINUITY VIA GRILLS

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Abstract. The idea of grill on a topological space was first introduced by Choquet [3] in 1947. It is observed from literature that the concept of grills is a powerful supporting tool, like nets and filters, in dealing with many a topological concept quite effectively. The notion of a micro topology was introduced and studied by Chandrasekar [2] which was defined Micro closed, Micro open, Micro interior and Micro closure. In this article, we present and study some new classes of sets and give a new decomposition of micro continuity in terms of grills.

keywords : Grill, Grill topology, Micro topology, Micro grill topology, Φ_m-open, g_m-set, $g_m\Phi_m$-set.

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1. Introduction

The idea of grill on a topological space was first introduced by Choquet [3] in 1947. It is observed from literature that the concept of grills is a powerful supporting tool, like nets and filters, in dealing with many a topological concept quite effectively. A number of theories and features has been handled in [1, 9, 11]. It helps to expand the topological structure which is used to measure the description rather than quantity, such as love, intelligence, beauty, quality of education and etc. In [12], Roy and Mukherjee defined and studied a typical topology associated rather naturally to the existing topology and a grill on a given topological space. Nano topology explored by M. Lellis Thivagar and Carmel Richard [10] can be described as a collection of nano approximations, a non-empty finite universe and empty set for which equivalence classes are buliding blocks. It is named as nano topology because whatever may be the size of the universe it has at most five open sets. The notion of a micro topology was introduced and studied by Chandrasekar [2] which was defined Micro closed, Micro open, Micro interior and Micro closure. S. Ganesan [5] introduced and studied Micro regular open in micro topological spaces. S. Ganesan [7] introduced and studied new type of micro grill topological spaces via micro grills and $m_{\mathcal{G}}g$-closed sets. In this article is to introduce we present and study some new classes of sets and give a new decomposition of micro continuity in terms of grills.

2. Preliminaries

Definition 2.1. [3] *A collection $\mathcal{G}$ of a nonempty subsets of a topological space X is called a grill on X if (i) $A \in \mathcal{G}$ and $A \subseteq B \subseteq X \Rightarrow B \in \mathcal{G}$, (ii) A, $B \subseteq X$ and $A \cup B \in \mathcal{G} \Rightarrow A \in \mathcal{G}$ or $B \in \mathcal{G}$. For any point x of a topological space (X, τ), we shall let $\tau(x)$ denote the collection of all open neighbourhoods of x.*

Definition 2.2. [10]

If (U, $\tau_R(X)$) is the nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then

(1) *The nano interior of the set A is defined as the union of all nano open subsets contained in A and it is denoted by nint(A). That is, nint(A) is the largest nano open subset of A.*
(2) *The nano closure of the set A is defined as the intersection of all nano closed sets containing A and it is denoted by ncl(A). That is, ncl(A) is the smallest nano closed set containing A.*

Definition 2.3. [2] *Let (U, $\tau_R(X)$) be a nano topological space. Then, $\mu_R(X) = \{N \cup (\acute{N} \cap \mu) : N, \acute{N} \in \tau_R(X)$ and $\mu \notin \tau_R(X)\}$ is called the Micro topology on U with respect to X. The triplet (U, $\tau_R(X)$, $\mu_R(X)$) is called Micro topological space and the elements of $\mu_R(X)$ are called Micro open sets and the complement of a Micro open set is called a Micro closed set.*

Definition 2.4. [2] *The Micro topology $\mu_R(X)$ satisfies the following axioms*

(1) *U, $\phi \in \mu_R(X)$.*
(2) *The union of the elements of any sub-collection of $\mu_R(X)$ is in $\mu_R(X)$.*
(3) *The intersection of the elements of any finite sub collection of $\mu_R(X)$ is in $\mu_R(X)$.*

Then $\mu_R(X)$ is called the Micro topology on U with respect to X. The triplet (U, $\tau_R(X)$, $\mu_R(X)$) is called Micro topological spaces. The elements of $\mu_R(X)$ are called Micro open (briefly, m-open) sets and the complement of a Micro open sets is called a Micro closed (briefly, m-closed) sets.

Definition 2.5. [2] *The Micro interior of a set A is denoted by Micro-int(A) (briefly, m-int(A)) and is defined as the union of all Micro open sets contained in A. i.e., Mic-int(A)* $= \cup \{G : G$ *is Micro open and* $G \subseteq A\}$.

Definition 2.6. [2] *The Micro closure of a set A is denoted by Micro-cl(A) (briefly, m-cl(A)) and is defined as the intersection of all Micro closed sets containing A. i.e., Mic-cl(A)* $= \cap \{F : F$ *is Micro closed and* $A \subseteq F\}$.

Definition 2.7. [2] *For any two Micro sets A and B in a Micro topological space (U,* $\tau_R(X)$, $\mu_R(X)$*),*

(1) *A is a Micro closed set if and only if Mic-cl(A) = A.*
(2) *A is a Micro open set if and only if Mic-int(A) = (A).*
(3) $A \subseteq B$ *implies Mic-int(A)* $\subseteq$ *Mic-int(B) and Mic-cl(A)* $\subseteq$ *Mic-cl(B).*
(4) *Mic-cl(Mic-cl(A)) = Mic-cl(A) and Mic-int(Mic-int(A)) = Mic-int(A).*
(5) *Mic-cl* $(A \cup B) \supseteq$ *Mic-cl(A)* $\cup$ *Mic-cl(B).*
(6) *Mic-cl*$(A \cap B) \subseteq$ *Mic-cl(A)* $\cap$ *Mic-cl(B).*
(7) *Mic-int*$(A \cup B) \supseteq$ *Mic-int(A)* $\cup$ *Mic-int(B).*
(8) *Mic-int*$(A \cap B) \subseteq$ *Mic-int(A)* $\cap$ *Mic-int(B).*
(9) *Mic-cl*$(\mathcal{A}^C) = [Mic - int(A)]^C$.
(10) *Mic-int*$(A^C) = [Mic - cl(A)]^C$

Definition 2.8. *Let (U,* $\tau_R(X)$, $\mu_R(X)$*) be a micro topological space and* $A \subseteq U$. *Then,*

(1) *A is called Micro pre-open if* $A \subseteq$ *Mic-int(Mic-cl(A))* [2].
(2) *A is called Micro regular-open if A = Mic-int(Mic-cl(A))* [5, 6].

The complement of above mentioned micro open sets are called their respective micro closed sets.

Definition 2.9. [4] *A subset A of a space* $(U, \tau_R(X), \mu_R(X))$ *is said to be*

(1) *mt-set if Mic-int(Mic-cl(A)) = Mic-int(A).*

(2) *mB-set if* $H = S \cap G$ *where S is Micro open and G is mt-set.*

Definition 2.10. *A map* $f : (U, \tau_R(X), \mu_R(X)) \to (L, \mu'_R(Y))$ *is said to be Micro continuous* [2] *(resp. mB-continuous* [4]*) if* $f^{-1}(V)$ *is an m-open (resp. mB-set) in U for every m-open set V of L.*

Let (K, $\mathcal{N}$, $\mathcal{M}$) be a micro topological space, where $\mathcal{N} = \tau_R(X)$ and $\mathcal{M} = \mu_R(X)$ and it is denoted by (K, $\mathcal{M}$).
Let (L, $\mathcal{N}'$, $\mathcal{M}'$) be a micro topological space, where $\mathcal{N}' = \tau'_R(Y)$ and $\mathcal{M}' = \mu'_R(Y)$ and is denoted by (L, $\mathcal{N}'$, $\mathcal{M}'$) or (L, $\mathcal{M}'$).
Let (K, $\mathcal{N}$, $\mathcal{M}$) be a micro topological space and $\mathcal{G}$ be a grill on K is called a micro grill topological space and it is denoted by (K, $\mathcal{N}$, $\mathcal{M}$, $\mathcal{G}$) or (K, $\mathcal{M}$, $\mathcal{G}$).
Let a space K we shall mean a micro grill topological spaces (K, $\mathcal{N}$, $\mathcal{M}$, $\mathcal{G}$). Also, the power set of K will be written as $\wp$(K), we shall let $\mathcal{M}(k)$ to stand for the collection of all micro open neighbourhoods of k.

Definition 2.11. [7] *Let (K,* $\mathcal{N}$*,* $\mathcal{M}$*) be a micro topological space and* $\mathcal{G}$ *be a grill on K. We define a mapping* $\Phi_m : \wp(K) \to \wp(K)$*, denoted by* $\Phi_{m\mathcal{G}}(A, \mathcal{M})$ *(for* $A \in \wp(K)$*) or* $\Phi_{m\mathcal{G}}(A)$ *or simply by* $\Phi_m(A)$*(when it is known which micro topology and grill on K we are talking about), called the operator associated with the grill* $\mathcal{G}$ *and the micro topology* $\mathcal{M}$*, and is defined by* $\Phi_m(A) = \Phi_{m\mathcal{G}}(A, \mathcal{M}) = \{k \in K : U \cap A \in \mathcal{G}, \forall\ U \in \mathcal{M}(k)\}$.

Note 2.12. [7] *$(K, \mathcal{N}, \mathcal{M})$ be a micro topological space with a grill $\mathcal{G}$ on K and for every A , B be subsets of K. Then*

(1) $A \subseteq B\ (\subseteq K) \Rightarrow \Phi_m(A) \subseteq \Phi_m(B)$,

(2) $\mathcal{G}_1 \subseteq \mathcal{G}_2 \Rightarrow \Phi_{m\mathcal{G}1}(A) \subseteq \Phi_{m\mathcal{G}2}(A)$ *(if $\mathcal{G}_1$ and $\mathcal{G}_2$ are two grills on K),*

(3) *If $A \notin \mathcal{G}$ then $\Phi_m(A) = \phi$.*

Proposition 2.13. [7] *Let $(K, \mathcal{N}, \mathcal{M})$ be a micro topological space and a grill $\mathcal{G}$ on K. Then for all $A, B \subseteq K$.*

(1) $\Phi_m(A \cup B) = \Phi_m(A) \cup \Phi_m(B)$,

(2) $\Phi_m(\Phi_m(A)) \subseteq \Phi_m(A) = m\text{-}cl(\Phi_m(A)) \subseteq m\text{-}cl(A)$.

Theorem 2.14. [7] *Let $\mathcal{G}$ be a grill on a micro topological spaces $(K, \mathcal{N}, \mathcal{M})$.*

(1) *If $U \in \mathcal{M}$ then $U \cap \Phi_m(A) = U \cap \Phi_m(U \cap A)$, for any $A \subseteq K$.*

(2) *If $\mathcal{M} \setminus \{\phi\} \subseteq \mathcal{G}$, then for all $U \in \mathcal{M}$, $U \subseteq \Phi_m(U)$.*

(3) $\Phi_m(A) \setminus \Phi_m(B) = \Phi_m(A \setminus B) \setminus \Phi_m(B)$, *for any $A, B \subseteq K$*

Corollary 2.15. [7] *Let $\mathcal{G}$ be a grill on a space $(K, \mathcal{N}, \mathcal{M})$ and suppose $A, B \subseteq K$, with $B \notin \mathcal{G}$. Then $\Phi_m(A \cup B) = \Phi_m(A) = \Phi_m(A \setminus B)$.*

Definition 2.16. [7] *Let $\mathcal{G}$ be a grill on a space $(K, \mathcal{N}, \mathcal{M})$. We define a map $\Psi_m : \wp(K) \to \wp(K)$ by $\Psi_m(A) = A \cup \Phi_m(A)$, for all $A \in \wp(K)$.*

Theorem 2.17. [7] *The above map Ψ_m satisfies the following conditions:*

(1) $A \subseteq \Psi_m(A), \forall\ A \subseteq K$,

(2) $\Psi_m(\phi) = \phi$ *and* $\Psi_m(k) = K$,

(3) *If $A \subseteq B\ (\subseteq K)$, then $\Psi_m(A) \subseteq \Psi_m(B)$,*

(4) $\Psi_m(A \cup B) = \Psi_m(A) \cup \Psi_m(B)$,

(5) $\Psi_m(\Psi_m(A)) = \Psi_m(A)$.

Now $\mathcal{M}^*$ $(\mathcal{G}, \mathcal{M}) = \{ U \subset K ; \Psi_m (K \setminus U) = K \setminus U\}$, *where for any* $A \subseteq K$, $\Psi_m(A) = A \cup \Phi_m(A) = m_{\mathcal{G}}\text{-}cl(A)$. $\mathcal{M}^*$ $(\mathcal{G}, \mathcal{M})$ *is called* $m_{\mathcal{G}}$*-topology which is finer than* $\mathcal{M}$ *(we simply write* $\mathcal{M}^*$ *for* $\mathcal{M}^*$ $(\mathcal{G}, \mathcal{M})$*. The elements of* $\mathcal{M}^*$ $(\mathcal{G}, \mathcal{M})$ *are called* $m_{\mathcal{G}}$*-open set and the complement of an* $m_{\mathcal{G}}$*-open set is called is called* $m_{\mathcal{G}}$*-closed set. Here* $m_{\mathcal{G}}\text{-}cl(A)$ *and* $m_{\mathcal{G}}\text{-}int(A)$ *will denote the closure and interior of A in* $(K, \mathcal{M}^*)$.

Definition 2.18. [7] *A basis* $\beta(\mathcal{G}, \mathcal{M})$ *for* $\mathcal{M}^*$ *can be described as follows:* $\beta(\mathcal{G}, \mathcal{M}) = \{ V \setminus F : V \in \mathcal{M}, F \notin \mathcal{G}\}$.

Theorem 2.19. [7] *Let* $(K, \mathcal{N}, \mathcal{M})$ *be a micro topological space and* $\mathcal{G}$ *be a grill on K. Then* $\beta(\mathcal{G}, \mathcal{M})$ *is a basis for* $\mathcal{M}^*$.

Corollary 2.20. [7] *For any grill* $\mathcal{G}$ *on a micro topological space* $(K, \mathcal{N}, \mathcal{M})$, $\mathcal{M} \subseteq \beta(\mathcal{G}, \mathcal{M}) \subseteq m_{\mathcal{G}}$.

Lemma 2.21. [7] *(1) If* $\mathcal{G}_1$ *and* $\mathcal{G}_2$ *are two grills on a space K with* $\mathcal{G}_1 \subseteq \mathcal{G}_2$, *then* $m_{\mathcal{G}2} \subseteq m_{\mathcal{G}1}$,
(2) If $\mathcal{G}$ *is a grill on a space K and* $B \notin \mathcal{G}$, *then B is* $m_{\mathcal{G}}$*-closed in* $(K, \mathcal{M}^*)$,
(3) For any subset A of a space K and any grill $\mathcal{G}$ *on K,* $\Phi_m(A)$ *is* $m_{\mathcal{G}}$*-closed.*

Definition 2.22. [7] *A subset A of a micro grill topological space* $(K, \mathcal{N}, \mathcal{M}, \mathcal{G})$ *is* $m_{\mathcal{G}}$ *dense in itself (resp.* $m_{\mathcal{G}}$*-perfect,* $m_{\mathcal{G}}$*-closed) if* $A \subseteq \Phi_m(A)$ *(resp.* $A = \Phi_m(A)$, $\Phi_m(A) \subseteq A$*).*

3. Some new classes of sets

Definition 3.1. *Let* $(K, \mathcal{N}, \mathcal{M})$ *be a micro topological space and* $\mathcal{G}$ *be a grill on K. A subset A in K is said to be*

(1) Φ_m*-open if* $A \subset m\text{-}int(\Phi_m(A))$ [8],

(2) g_m*-set if m-int(*Ψ_m*(A)) = m-int(A),*

(3) $g_m\Phi_m$*-set if m-int(*Φ_m*(A)) = m-int(A).*

Definition 3.2. [8] *Let (K,* $\mathcal{N}$*,* $\mathcal{M}$*) be a micro topological space and* $\mathcal{G}$ *be a grill on K. A subset A in K is said to be*

(1) $\mathcal{G}$*-m-pre-open if A* $\subset$ *m-int(*Ψ_m*(A)),*

(2) $\mathcal{G}$*-m-regular-open if m-int(*Ψ_m*(A)) = A.*

Remark 3.3. *(1) m-open set and* Φ_m*-open set are independent from each other* [8]. *(2) Every* $g_m\Phi_m$*-set is a* g_m*-set, but it is not conversely.*

Example 3.4. *Let* $K = \{a, b, c\}$ *with* $K / R = \{\{b\}, \{a, c\}\}$ *and* $X = \{b\}$*. The nano topology* $\mathcal{N} = \{\phi, \{b\}, K\}$*. If* $\mu = \{b, c\}$ *then the micro topology* $\mathcal{M} = \{\phi, \{b\}, \{b, c\}, K\}$ *and* $\mathcal{G} = \{\{a\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, K\}$*. Then* g_m*-set are* ϕ*, K,* $\{a\}$*,* $\{b\}$*,* $\{c\}$*,* $\{a, b\}$*,* $\{a, c\}$ *;* $g_m\Phi_m$*-set are* ϕ*, K,* $\{a\}$*,* $\{c\}$*,* $\{a, c\}$*. It is clear that* $\{a, b\}$ *is* g_m*-set but it is not* $g_m\Phi_m$*-set.*

Proposition 3.5. $m_{\mathcal{G}}$*-closed set is equivalent to a* g_m*-set.*

Proof. Let A be a subset in (K, $\mathcal{M}$, $\mathcal{G}$). Then Φ_m(A) is $m_{\mathcal{G}}$-closed by Lemma 2.21 (3). m-int(Ψ_m(Φ_m(A))) = m-int(Φ_m(A) $\cup$ Φ_m(Φ_m(A))) = m-int(Φ_m(A))[by Proposition 2.13 (2)], i.e. Φ_m(A) is a g_m-set.

Proposition 3.6. *Every* $\mathcal{G}$*-m-regular-open set is a* g_m*-set*

Proof. Obvious.

Example 3.7. *Let* $K = \{a, b, c\}$ *with* $K / R = \{\{a, b, c\}\}$ *and* $X = \{b, c\}$*. The nano topology* $\mathcal{N} = \{\phi, K\}$*. If* $\mu = \{a\}$ *then the micro topology* $\mathcal{M} = \{\phi, \{a\}, K\}$ *and* $\mathcal{G} = \{\{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, K\}$*. Then* g_m*-set are* ϕ*, K,* $\{b\}$*,* $\{c\}$*,* *{b, c} ; $\mathcal{G}$-m-regular-open set are ϕ, K. It is clear that {b, c} is g_m-set but it is not $\mathcal{G}$-m-regular-open set.*

Proposition 3.8. *Every mt-set is a g_m-set but not conversely.*

Proof. Let A be a mt-set. Then m-int(A) $\subset$ m-int(Ψ_m(A)) = m-int(A $\cup$ Φ_m(A)) $\subset$ m-int(A $\cup$ m-cl(A))[by Proposition 2.13 (2)] = m-int(m-cl(A)) = m-int(A). Therefore, A is a g_m-set.

Example 3.9. *Let K = {a, b, c} with K / R= {{a}, {b, c}} and X= {a}. The nano topology $\mathcal{N}$ = {ϕ, {a}, K}. If μ = {b, c} then the micro topology $\mathcal{M}$ = {ϕ, {a}, {b, c}, K} and $\mathcal{G}$ = {{a}, {b}, {a, b}, {a, c}, {b, c}, K}. Then g_m-set are ϕ, K, {a}, {c}, {a, c}, {b, c} ; mt-sets are ϕ, K, {a}, {b, c}. It is clear that {a, c} is g_m-set but it is not mt-set.*

Proposition 3.10. *If A, B are two g_m-sets, then A $\cap$ B is a g_m-set.*

Proof. m-int(A$\cap$B) $\subset$ m-int(Ψ_m(A $\cap$ B)) = m-int(Ψ_m(A $\cap$ B) $\cap$ Ψ_m(A $\cap$ B)) = m-int(Ψ_m(A $\cap$ B)) $\cap$ m-int(Ψ_m(A $\cap$ B)) $\subset$ m-int(Ψ_m(A)) $\cap$ m-int(Ψ_m(B)) = m-int(A) $\cap$ m-int(B) = m-int(A $\cap$ B). Then A $\cap$ B is a g_m-set

Proposition 3.11. [8] *Every $\mathcal{G}$-m-pre-open set A is a Micro pre-open set.*

Remark 3.12. *By [Example 3.14, [7]], since if $\mathcal{G}$ = $\wp$(K) \ {ϕ} in (K, $\mathcal{N}$, $\mathcal{M}$), then $\mathcal{M}$ = $m_{\mathcal{G}}$, $\mathcal{G}$-m-pre-open and Micro pre-open set are equivalent.*

Proposition 3.13. *If A is a $\mathcal{G}$-m-pre-open, then m-cl(m-int(Ψ_m(A))) = m-cl(A).*

Proof. m-cl(A) $\subset$ m-cl(m-int(Ψ_m(A))) $\subset$ m-cl(Ψ_m(A)) = m-cl(A $\cup$ Φ_m(A)) = m-cl(A) $\cup$ m-cl(Φ_m(A)) = m-cl(A) $\cup$ Φ_m(A) = m-cl(A) $\cup$ m-cl(A) [by Proposition 2.13 (2)] $\subset$ m-cl(A).

Proposition 3.14. [8] *Every Φ_m-open set A is $\mathcal{G}$-m-pre-open.*

Proposition 3.15. *Let (K, $\mathcal{M}$, $\mathcal{G}$) be a micro grill topological space with $\mathcal{I}$ arbitrary index set. Then:*

(1) *If $\{A_i|i\in \mathcal{I}\}$ are $\mathcal{G}$-m-pre-open sets, then $\cup \{A_i|i\in \mathcal{I}\}$ is a $\mathcal{G}$-m-pre-open set.*
(2) *If A is a $\mathcal{G}$-m-pre-open set and $U \in \mathcal{M}$, then ($A \cap U$) is a $\mathcal{G}$-m-pre-open set.*

Proof. (1) Let $\{\mathrm{A}_i|i\in \mathcal{I}\}$ be $\mathcal{G}$-m-pre-open sets, then $\mathrm{A}_i\subset$ m-int($\Psi_m(\mathrm{A}_i)$) for every i $\in \mathcal{I}$. Thus $\cup$ $\mathrm{A}_i\subset \cup$ (m-int($\Psi_m(\mathrm{A}_i)$))$\subset$ m-int($\cup$ ($\Psi_m(\mathrm{A}_i)$)) $= m - int(\cup\ (\mathrm{A}_i\cup \Phi_m(\mathrm{A}_i))) = m - int(\cup\ \mathrm{A}_i)\cup\ (\cup\ \Phi_m(\mathrm{A}_i)))) = m - int(\cup\ \mathrm{A}_i\cup\ \Phi_m(\cup\ \mathrm{A}_i)) = m - int(\Psi_m(\cup\ \mathrm{A}_i))$.

(2) Let A be a $\mathcal{G}$m-pre-open set and U $\in \mathcal{M}$ [by Theorem 2.13 (1)], U $\cap$ A $\subset$ U $\cap$ m-int(Ψ_m(A)) = U $\cap$ m-int(A $\cup$ Φ_m(A)) = m-int(U $\cap$ (A $\cup$ Φ_m(A))) = m-int(U $\cap$ A) $\cup$ (U $\cap$ Φ_m(A))) = m-int(U $\cap$ A) $\cup$ (U $\cap$ Φ_m(U $\cap$ A))) $\subset$ m-int((U $\cap$ A) $\cup$ Φ_m(U $\cap$ A)) = m-int(Ψ_m(U $\cap$ A)), which completes the proof.

Definition 3.16. *Let (K, $\mathcal{M}$) be a micro topological space and $\mathcal{G}$ a grill on K. A subset A in K is said to be*

(1) *a $\mathcal{G}_m$-set if $H = S \cap G$ where S is m-open and G is a g_m-set.*
(2) *a $\mathcal{G}_m\Phi_m$-set if $H = S \cap G$ where S is m-open and G is a $g_m\Phi_m$-set.*

Proposition 3.17. *(1) Every g_m-set is a $\mathcal{G}_m$-set but not conversely.*
(2) Every $g_m\Phi_m$-set is a $\mathcal{G}_m\Phi_m$-set but not conversely.

Proof. Obvious.

Proposition 3.18. *An m-open set U is a $\mathcal{G}_m$-set (resp. $\mathcal{G}_m\Phi_m$-set).*

Proof. U = U $\cap$ K, m-int(Ψ_m(K)) = m-int(K).

Proposition 3.19. *Every $m_{\mathcal{G}}$-closed set is a $\mathcal{G}_m$-set*

Proof. It follows from Proposition 3.5 and Proposition 3.17 (1).

Proposition 3.20. *(1) Every mB-set is a $\mathcal{G}_m$-set.*

(2) Every $\mathcal{G}_m\Phi_m$-set is a $\mathcal{G}_m$-set.

Proof. (1) Let H be a mB-set. Then H = S $\cap$ G, where S $\in$ m-open and G is a mt-set. H = S $\cap$ m-int(G) = S $\cap$ m-int(m-cl(G)) = S $\cap$ m-int(G $\cup$ m-cl(G)) $\supset$ U $\cap$ m-int(G $\cup$ Φ_m(G)) = U $\cap$ m-int(Ψ_m(G)) $\supset$ S $\cap$ m-int(G) = H. Therefore H is a $\mathcal{G}_m$-set.

(2) Let H be a $\mathcal{G}_m\Phi_m$-set. Then H = S $\cap$ G, where S $\in$ m-open and G is a $\mathcal{G}_m\Phi_m$-set. H = S $\cap$ m-int(G) = S $\cap$ m-int(Φ_m(G)) $\supset$ S $\cap$ m-int(G $\cup$ Φ_m(G)) = S $\cap$ m-int(Ψ_m(G)) $\supset$ S $\cap$ m-int(G) = H. Therefore H is a $\mathcal{G}_m$-set.

The converse of Proposition 3.20 is false as it is shown by the following example.

Example 3.21. *Let (K, $\mathcal{N}$, μ, $\mathcal{M}$, $\mathcal{G}$) be defined as an Example 3.4. Then $\mathcal{G}_m$-sets are ϕ, K, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, $\{b, c\}$; $\mathcal{G}_m\Phi_m$-sets are ϕ, K, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, c\}$, $\{b, c\}$; mB-sets are ϕ, K, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, c\}$, $\{b, c\}$. It is clear that $\{a, b\}$ is $\mathcal{G}_m$-set but it is not mB-set. Also it is clear that $\{a, b\}$ is $\mathcal{G}_m$-set but is not $\mathcal{G}_m\Phi_m$-set.*

Proposition 3.22. *A subset J in a space (K, $\mathcal{M}$, $\mathcal{G}$) is m-open if and only if it is a $\mathcal{G}_m$-pre-open and a $\mathcal{G}_m$-set.*

Proof. Necessity. It follows from Proposition 3.18 and the obvious fact that every m-open set is $\mathcal{G}_m$-pre-open. Sufficiency. Since J is a $\mathcal{G}_m$-set, then J = E $\cap$ F where

E is an m-open set and m-int(Ψ_m(F)) = m-int(F). Since J is also $\mathcal{G}_m$-preopen, we have J $\subset$ m-int(Ψ_m(J)) = m-int(Ψ_m(E $\cap$ F)) = m-int(Ψ_m(E $\cap$ F) $\cap$ Ψ_m(E $\cap$ F)) $\subset$ m-int(Ψ_m(E) $\cap$ Ψ_m(F)) = m-int(Ψ_m(E) $\cap$ m-int(Ψ_m(F)) = m-int(E $\cup$ Φ_m(E)) $\cap$ m-int(Ψ_m(F)) $\subset$ m-int(m-cl(E)) $\cap$ m-int(Ψ_m(F)) = m-int(m-cl(E)) $\cap$ m-int(F). Hence J = E $\cap$ F = (E $\cap$ F) $\cap$ E $\subset$ (m-int(m-cl(E) $\cap$ m-int(F)) $\cap$ E = (m-int(m-cl(E)) $\cap$ E) $\cap$ m-int(F) = E $\cap$ m-int(F). Therefore, J = E $\cap$ F $\supset$ E $\cap$ m-int(F) and J = E $\cap$ m-int(F). Thus J is an m-open set.

Corollary 3.23. *If S is both $\mathcal{G}_m\Phi_m$-set and Φ_m-open set in (K, $\mathcal{M}$, $\mathcal{G}$), then S is m-open.*

Remark 3.24. *$\mathcal{G}_m$-pre-open set and $\mathcal{G}_m$-set are independent from each other as in the following Example.*

Example 3.25. *Let K = $\{a, b, c\}$ with K / R= $\{\{a, b, c\}\}$ and X= $\{b, c\}$. The nano topology $\mathcal{N}$ = $\{\phi, K\}$. If $\mu = \{a, b\}$ then the micro topology $\mathcal{M}$ = $\{\phi, \{a, b\}, K\}$ and $\mathcal{G}$ = $\{\{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, K\}$. Then $\mathcal{G}_m$-pre-open set are ϕ, K, $\{a\}$, $\{b\}$, $\{a, b\}$, $\{a, c\}$, $\{b, c\}$; $\mathcal{G}_m$-set are ϕ, K, $\{c\}$, $\{a, b\}$. Here $\{b, c\}$ is $\mathcal{G}_m$-pre-open set but it is not $\mathcal{G}_m$-set. Also it is clear that $\{c\}$ is $\mathcal{G}_m$-set but it is not $\mathcal{G}_m$-pre-open set.*

Definition 3.26. *Let (K, $\mathcal{M}$, $\mathcal{G}$) be a grill space and A $\subset$ K. A set A is said to be $\mathcal{G}$-m-dense in K, if Ψ_m(A) = K.*

Proposition 3.27. *A subset A of a grill $\mathcal{G}$ in a space (K, $\mathcal{M}$, $\mathcal{G}$) is $\mathcal{G}$-dense if and only if for every Micro open set U containing $k \in K$, A $\cap$ U $\in \mathcal{G}$.*

Proof. Necessity. Let A be a $\mathcal{G}$-m-dense set. Then, for every m-open set U containing k in a space K, k $\in$ Ψ_m(A) = A $\cup$ Φ_m(A). Hence if k $\in$ A, then A $\cap$ U $\in \mathcal{G}$ and if k

$\in \Phi_m$(A), then A $\cap$ U $\in \mathcal{G}$.

Sufficiency. Let every k $\in$ K. Moreover, let every Micro open subset U of K containing k such that A $\cap$ U $\in \mathcal{G}$. Then if k $\in$ A or k $\in \Phi_m$(A), we have A $\cap$ U $\in \mathcal{G}$. It follows that k $\in \Psi_m$(A) and thus K $\subset \Psi_m$(A). Therefore Ψ_m(A) = K.

Proposition 3.28. *If U is an Micro open set and A is a $\mathcal{G}$-m-dense set in (K, $\mathcal{M}$, $\mathcal{G}$), then $\Psi_m(U) = \Psi_m(U \cap A)$.*

Proof. Since A $\cap$ U $\subset$ U, we have Ψ_m(U $\cap$ A) $\subset \Psi_m$(U). Conversely, if k $\in \Psi_m$(U), k $\in$ U and x $\in \Phi_m$(U). Then for every Micro open set V containing k, U $\cap$ V $\in \mathcal{G}$. Put W = U $\cap$ V $\in \mathcal{M}(k)$. Since Ψ_m(A) = K, W $\cap$ A $\in \mathcal{G}$, i.e. W = (U $\cap$ A) $\cap$ V $\in \mathcal{G}$. Therefore, k $\in \Psi_m$(U $\cap$ A) and Ψ_m(U) = Ψ_m(U $\cap$ A).

Proposition 3.29. *For any subset A of a space (K, $\mathcal{M}$, $\mathcal{G}$), the following are equivalent:*

(1) *A is $\mathcal{G}$-m-pre-open,*
(2) *there is a $\mathcal{G}$-m-regular open set U of K such that $A \subset U$ and $\Psi_m(A) = \Psi_m(U)$,*
(3) *A is the intersection of $\mathcal{G}$-m-regular-open set and a $\mathcal{G}$-m-dense set,*
(4) *A is the intersection of an m-open set and a $m\mathcal{G}_m$-dense set.*

Proof. (1) $\Rightarrow$ (2): Let A be $\mathcal{G}$-m-pre-open in (K, $\mathcal{M}$, $\mathcal{G}$), i.e. A $\subset$ m-int(Ψ_m(A)). Let U = m-int(Ψ_m(A)). Then U is $\mathcal{G}$-m-regular-open such that A $\subset$ U and Ψ_m(A) $\subset$ Ψ_m(U) = Ψ_m(m-int(Ψ_m(A)) $\subset \Psi_m(\Psi_m$(A))[by Theorem 2.17 (5)] = Ψ_m(A). Hence Ψ_m(A) = Ψ_m(U).

(2) $\Rightarrow$ (3): Suppose (2) holds. Let D = A $\cup$ (K $\setminus$ U). Then D is a $\mathcal{G}$-m-dense set. In fact Ψ_m(D) = Ψ_m(A $\cup$ (K $\setminus$ U)) = Ψ_m(A) $\cup$ Ψ_m(K $\setminus$ U) = Ψ_m(U) $\cup$ Ψ_m(K $\setminus$ U) = Ψ_m(U $\cup$ (K $\setminus$ U)) = Ψ_m(K) = K [by Theorem 2.17 (2)]. Therefore, A = D $\cap$ U, D is a $\mathcal{G}$-m-dense set and U is a $\mathcal{G}$-m-regular-open set.

(3) $\Rightarrow$ (4): Every $\mathcal{G}$-m-regular-open set is m-open.

(4) $\Rightarrow$ (1): Suppose A = U $\cap$ D with U and D $m\mathcal{G}_m$-dense. Then Ψ_m(A) = Ψ_m(U) since A = U $\cap$ D, Ψ_m(A) = Ψ_m(U $\cap$ D) = Ψ_m(U). Hence A $\subset$ U $\subset$ Ψ_m(U) = Ψ_m(A), that is, A $\subset$ m-int(Ψ_m(A)).

Proposition 3.30. *If A is both Micro regular-open and $\mathcal{G}$-m-pre-open set in (K, $\mathcal{M}$, $\mathcal{G}$), then it is $\mathcal{G}$-m-regular-open.*

Proof. A $\subset$ m-int(Ψ_m(A)) = m-int(A $\cup$ Φ_m(A)) $\subset$ m-int(A $\cup$ m-cl(A)) $\subset$ m-int(m-cl(A)) = A.

Remark 3.31. *It should be noted that m-open sets and g_m-sets are independent and Micro regular open sets and $\mathcal{G}$-m-regular-open sets are also independent. Every $\mathcal{G}$-m-regular-open set is m-open. Micro Regular openness implies Micro openness and $\mathcal{G}$-m-regular-open sets imply g_m-sets.*

4. Decomposition of continuity

Definition 4.1. *A map f : (K, $\mathcal{M}$, $\mathcal{G}$) $\rightarrow$ (L, $\mathcal{M}'$) is called:*

(1) *Φ_m-continuous if f^{-1}(V) is an Φ_m-open in K for every m-open set V of L.*
(2) *$\mathcal{G}_m\Phi_m$-continuous if f^{-1}(V) is an $\mathcal{G}_m\Phi_m$-set in K for every m-open set V of L.*
(3) *$\mathcal{G}_m$-continuous if f^{-1}(V) is an $\mathcal{G}_m$-set in K for every m-open set V of L.*
(4) *$\mathcal{G}$-m-pre-continuous if f^{-1}(V) is an $\mathcal{G}$-m-pre-open in K for every m-open set V of L.*

Theorem 4.2. *(1) Every mB-continuous is a $\mathcal{G}_m$-continuous.*
(2) Every $\mathcal{G}_m\Phi_m$-continuous is a $\mathcal{G}_m$-continuous.

Proof. It follows from Proposition 3.20.

Remark 4.3. *$\mathcal{G}$-m-pre-continuous and $\mathcal{G}_m$-continuous are independent from each other as in the following exapmle.*

Example 4.4. *Let (K, $\mathcal{N}$, μ, $\mathcal{M}$, $\mathcal{G}$) be defined as an Example 3.25. Let L = {a, b, c} with L / R'= {{c}, {a, b}} and Y = {c}. Then nano topology $\tau'_R(Y)$ = {ϕ, {c}, L}. If μ = {a, b} then the micro topology $\mathcal{M}$ = {ϕ, {c}, {a, b}, K}. Define f : (K, $\mathcal{M}$, $\mathcal{G}$) $\rightarrow$ (L, $\mathcal{M}'$) be the identity map. Then f is $\mathcal{G}_m$-continuous but not $\mathcal{G}$-m-pre-continuous, since f^{-1}({c})= {c} is not $\mathcal{G}$-m-pre open in (K, $\mathcal{M}$, $\mathcal{G}$).*

Example 4.5. *Let (K, $\mathcal{N}$, μ, $\mathcal{M}$, $\mathcal{G}$) be defined as an Example 3.25. Let L = {a, b, c} with L / R'= {{a}, {b, c}} and Y = {a}. Then nano topology $\tau'_R(Y)$ = {ϕ, {a}, L}. If μ = {b, c} then the micro topology $\mathcal{M}$ = {ϕ, {a}, {b, c}, K}. Define f : (K, $\mathcal{M}$, $\mathcal{G}$) $\rightarrow$ (L, $\mathcal{M}'$) be the identity map. Then f is $\mathcal{G}$-m-pre-continuous but not $\mathcal{G}_m$-continuous, since f^{-1}({b, c})= {b, c} is not $\mathcal{G}_m$-set in (K, $\mathcal{M}$, $\mathcal{G}$).*

Theorem 4.6. *A map f : (K, $\mathcal{M}$, $\mathcal{G}$) $\rightarrow$ (L, $\mathcal{M}'$) is Micro-continuous if and only if it is both $\mathcal{G}$-m-pre-continuous and $\mathcal{G}_m$-continuous.*

Proof. It follows from Proposition 3.22.

Theorem 4.7. *A map f : (K, $\mathcal{M}$, $\mathcal{G}$) $\rightarrow$ (L, $\mathcal{M}'$) is both Φ_m-continuous and $\mathcal{G}_m\Phi_m$-continuous then S is Micro-continuous.*

Proof. It follows from Corollary 3.23.

Conclusion: We presented several definitions, properties, explanations and examples inspired from the concept of micro continuity in terms of grills.

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